Pg 251, # 8, 11, 13, 20, 31, 42

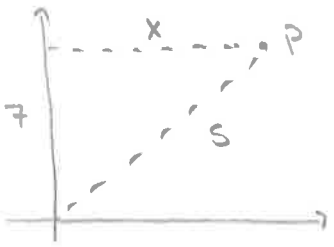
(8) $A = \pi r^2$, $\frac{dr}{dt} = .01$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(50)(.01)$

$\frac{dA}{dt} = \pi$ cm²/sec

(13)



$\frac{ds}{dt} = 300$

when $s = 10$,

$x = \sqrt{51}$

$7^2 + x^2 = s^2$

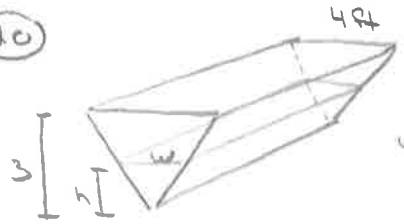
$0 + 2x\frac{dx}{dt} = 2s\frac{ds}{dt}$

$\frac{dx}{dt} = \frac{s\frac{ds}{dt}}{x}$

$= \frac{10(300)}{\sqrt{51}}$

$\frac{dx}{dt} \approx 420$ mph

(20)



$\frac{dV}{dt} = 2.5$ ft³/min

want $\frac{dh}{dt}$ when $h = 2$

$V = \frac{1}{2}wh(15)$

$\frac{4}{3} = \frac{w}{h}$

$4h = 3w$

$w = \frac{4}{3}h$

$V = \frac{15}{2} \cdot \frac{4}{3}h^2$

$V = 10h^2$

$\frac{dV}{dt} = 20h\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{2.5}{20(2)}$

$\frac{dh}{dt} = \frac{1}{16}$ ft/min

(11)

$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

$\frac{dV}{dt} = 100\pi$ ft³/min

want $\frac{dr}{dt}$ when $r = 5$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

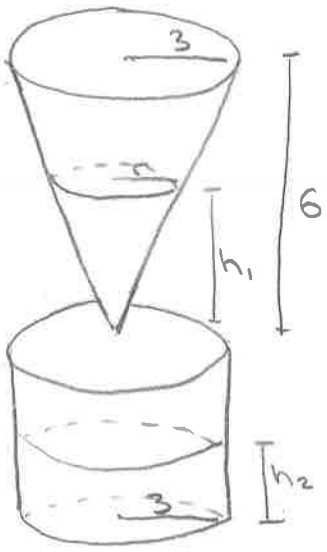
$\frac{dr}{dt} = \frac{100\pi}{100\pi} = 1$ ft/min

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$= 8(\pi)(5)(1)$

$\frac{dSA}{dt} = 40\pi$ ft²/min

(42)



a) want $\frac{dh_2}{dt}$

$V = 9\pi h_2$

$\frac{dV}{dt} = 9\pi \frac{dh_2}{dt}$

$10 = 9\pi \frac{dh_2}{dt}$

$\frac{dh_2}{dt} = \frac{10}{9\pi}$ in/min

b) Filter: want $\frac{dh_1}{dt}$

$V = \frac{1}{3}\pi r^2 h_1$

$\frac{6}{12} = \frac{h_1}{r}$

$r = \frac{h_1}{2}$

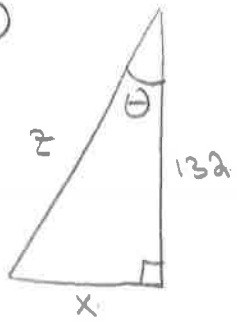
$V = \frac{1}{3}\pi \frac{1}{4} h_1^3$

$\frac{dV}{dt} = \frac{\pi}{4} h_1^2 \frac{dh_1}{dt}$

$10 = \frac{25\pi}{4} \frac{dh_1}{dt}$

$\frac{dh_1}{dt} = \frac{8}{5\pi}$ in/min

(31)



want $\frac{d\theta}{dt}$ when $x = 0$

$\frac{dx}{dt} = 264$ ft/sec

$\tan\theta = \frac{x}{132}$

$\theta = \tan^{-1}\left(\frac{x}{132}\right)$

$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{x}{132}\right)^2} \cdot \frac{1}{132} \cdot \frac{dx}{dt}$

$= 1\left(\frac{1}{132}\right)(264)$

a) $\frac{d\theta}{dt} = 2$ rad/sec when $x = 0$

b) when $t = \frac{1}{2}$, $x = 132$

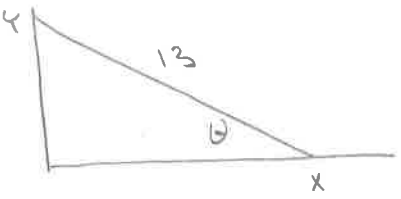
$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{132}{132}\right)^2} \cdot \frac{1}{132} \cdot \frac{dx}{dt}$

$\frac{d\theta}{dt} = 1$ rad/sec

$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$

Ps 251 # 19, 14, 17, 18, 22

(19)



when $x = 12$

$\frac{dx}{dt} = 5$ ft/sec

a) $\frac{dy}{dt} = ?$

$x^2 + y^2 = 13^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

when $x = 12$ ..

$12^2 + y^2 = 13^2$

$y = 5$

$\frac{dy}{dt} = \frac{-x\frac{dx}{dt}}{y} = \frac{-12 \cdot 5}{5}$

$\frac{dy}{dt} = -12$ ft/sec

b) $A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\left(x\cdot\frac{dy}{dt} + y\cdot\frac{dx}{dt}\right)$

$= \frac{1}{2}(12\cdot -12 + 5\cdot 5)$

$= \frac{1}{2}(-144 + 25)$

$\frac{dA}{dt} = -\frac{119}{2}$ ft²/sec

$= -59.5$

c) $\sin\theta = \frac{y}{13}$

when $x = 12$

$\cos\theta = \frac{12}{13}$

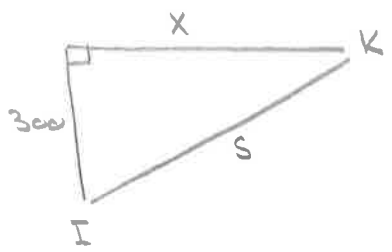
$\cos\theta\frac{d\theta}{dt} = \frac{1}{13}\cdot\frac{dy}{dt}$

$\frac{d\theta}{dt} = \frac{\frac{1}{13}\cdot(-12)}{\cos\theta}$

$= \frac{-\frac{12}{13}}{\frac{12}{13}}$

$\frac{d\theta}{dt} = -1$ rad/sec

(14)



$\frac{dx}{dt} = 25$, want $\frac{ds}{dt}$

when $s = 500$

$300^2 + x^2 = s^2$

$0 + 2x\frac{dx}{dt} = 2s\frac{ds}{dt}$

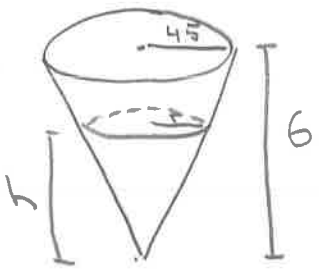
$\frac{ds}{dt} = \frac{x\frac{dx}{dt}}{s}$ when $s = 500$

$x = 400$

$\frac{ds}{dt} = \frac{400(25)}{500}$

$\frac{ds}{dt} = 20$ ft/sec

(17)



$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 50 \text{ m}^3/\text{s}$

want $\frac{dh}{dt}$ when $h = 5$

$\frac{r}{h} = \frac{4.5}{6}$

$r = \frac{4.5}{6} h$

a)

$V = \frac{1}{3}\pi \left(\frac{4.5}{6} h\right)^2 h$

$V = 18.75\,\pi\, h^3$

$\frac{dV}{dt} = 56.25\,\pi\, h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{50}{56.25 \cdot \pi \cdot (5)^2}$

$= .0113 \text{ m/min}$

$\frac{dh}{dt} = 1.13 \text{ cm/min}$

b) $r = \frac{4.5}{6} h$

$\frac{dr}{dt} = \frac{4.5}{6} \frac{dh}{dt}$

$\frac{dr}{dt} = -8.488 \text{ cm/min}$

(18)

a) want $\frac{dy}{dt}$ when $y = 8$.

$\frac{dV}{dt} = -6 \text{ m}^3/\text{min}$

$V = \frac{\pi}{3} y^2 (3R - y)$

$V = \frac{\pi}{3} y^2 (39 - y)$

$V = 13\pi y^2 - \frac{\pi}{3} y^3$

a) $\frac{dV}{dt} = 26\pi y \frac{dy}{dt} - \pi y^2 \frac{dy}{dt}$

$-6 = \left[26\pi \cdot 8 - \pi 8^2\right] \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{-6}{144\pi} = \frac{-1}{24\pi} =$

$\frac{dy}{dt} = -0.0133 \text{ m/min}$

b) $r^2 + (13 - y)^2 = 13^2$

$r = \sqrt{169 - (169 - 26y + y^2)}$

$r = \sqrt{26y - y^2}$

c) $r = (26y - y^2)^{1/2}$

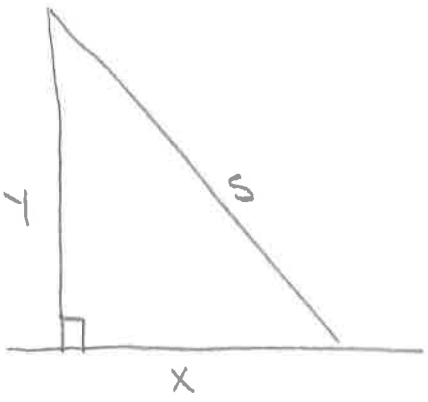
$\frac{dr}{dt} = \frac{1}{2}(26y - y^2)^{-1/2} (26 - 2y) \frac{dy}{dt}$

$= \frac{13 - y}{\sqrt{26y - y^2}} \cdot \frac{dy}{dt}$

$= \frac{13 - 8}{\sqrt{26 \cdot 8 - 8^2}} \cdot -\frac{1}{24\pi}$

$\frac{dr}{dt} = -0.005526 \text{ m/min}$

(22)



$\frac{dy}{dt} = 1$ ft/sec

$\frac{dx}{dt} = 17$ ft/sec

want $\frac{ds}{dt}$ when $t = 3$

at $t = 3$, $y = 68$

$x = 51$

$s = \sqrt{68^2 + 51^2} = 85$

$x^2 + y^2 = s^2$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2s \frac{ds}{dt}$

$51(17) + 68(1) = 85 \frac{ds}{dt}$

$\frac{ds}{dt} = 11$ ft/sec